18.06 Note Lecture 01

n Linear Equations, n Unknowns

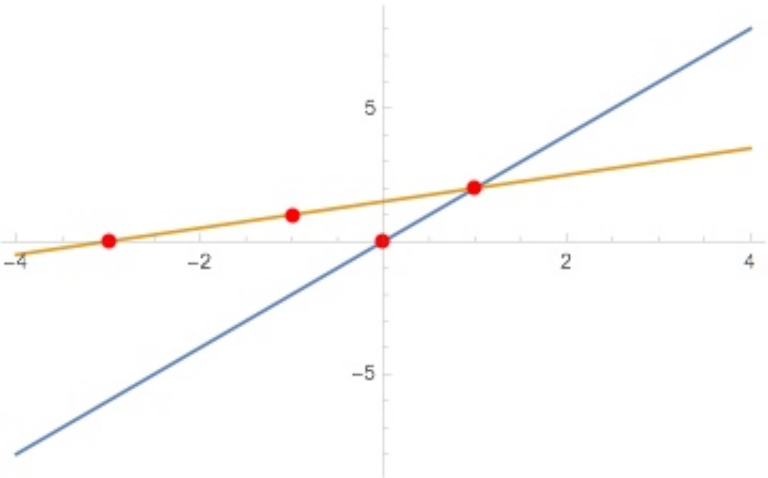
$$
\begin{aligned}
2x - y &= 0 \\
-x + 2y &= 3
\end{aligned}
$$

wrriten as

$$\underbrace{\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

A is the matrix of coefficients
X is the vector of unknowns

Row Picture

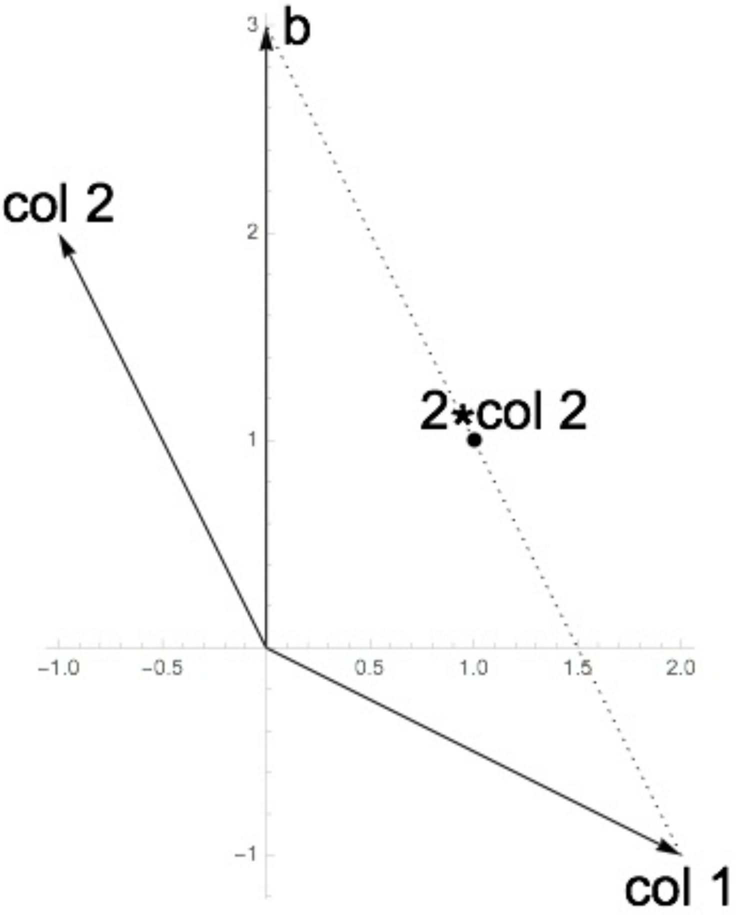


$(1, 2)$ is the solution

* Column Picture

$$x \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

This called **linear combination** of columns

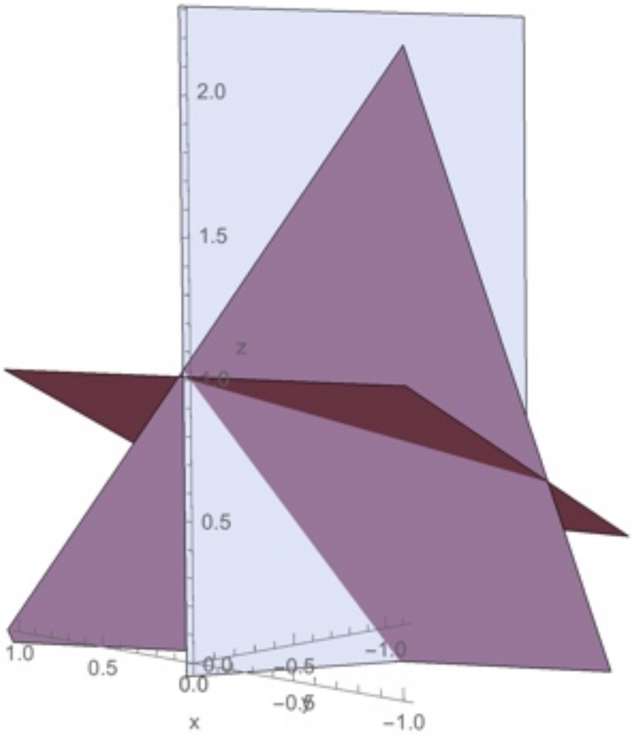


3D Case

$$
\begin{aligned}
2x - y &= 0 \\
-x + 2y - z &= -1 \\
-3y + 4z &= 4
\end{aligned}
$$

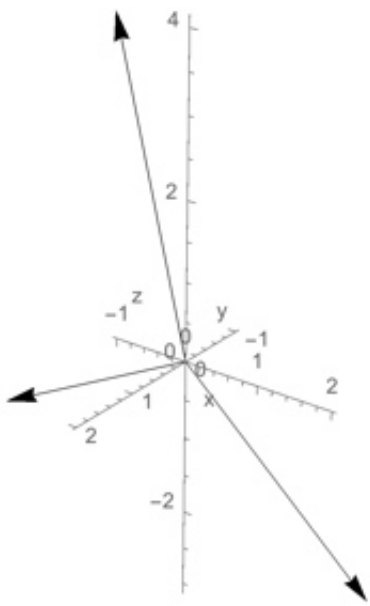
Row Picture

$$
A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}
$$



Column Picture

$$x\begin{bmatrix}2\\-1\\0\end{bmatrix}+y\begin{bmatrix}-1\\2\\-3\end{bmatrix}+z\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}0\\-1\\4\end{bmatrix}$$



The solution is $x = 0, y = 0, z = 1$

Question

Can solve $Ax = b$ for every b?

Do the linear combinations of the columns fill 3-D space?

For matrix A, answer is **YES**.

If 3 vectors happen to lie in the **same** plane, the vector which out of the plane is unreachable, so the answer is **NO**.

Matrix Form

Matrix Multiplication

$$Ax = b$$

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

Ax is a linear combination of columns of A.